## BOOK REVIEWS

E. H. Dooijes, explains the cascade-like wavelet algorithms from the viewpoint of two-channel and multi-channel subband coding schemes. Calculation of the Wavelet Decomposition using Quadrature Formulae, by W. Sweldens and R. Piessens, gives another introduction into multiresolution approximation, and it shows in detail how to approximate inner products of  $f \in L^2(\mathbb{R})$  with wavelets and scaling functions by combinations of point evaluations of f. Fast Wavelet Transforms and Calderón-Zygmund Operators, by T. H. Koornwinder, applies periodized wavelets to derive wavelet expansions of certain integral operators. The Finite Wavelet Transform with an Application to Seismic Processing, by J. A. H. Alkemade, and Wavelets Understand Fractals, by M. Hazewinkel, conclude the book with two further recent applications of wavelet analysis.

The book can be highly recommended to everyone who needs a quick account of wavelet theory as well as some ideas of wavelet applications. Results and basic theorems are stated in a rigorous and very satisfactory way, without overloading the treatment by including too many concisely worked-out proofs. Those interested in a more complete treatment will find enough hints where to look up the details. While not being a textbook for students at an intermediate level, it can be useful as an aide in more advanced courses or seminars. For specialists in the field, the book can serve as a nice reference work; engineers and other people interested in algorithms for the fast wavelet transform will find it a useful guide to go directly to their specific interests. I am convinced that this "elementary treatment of theory and applications" will become a standard reference for a broad audience.

KURT JETTER

S. B. YAKUBOVICH AND YU. F. LUCHKO, The Hypergeometric Approach to Integral Transforms and Convolutions, Mathematics and Its Applications 287, Kluwer Academic, Dordrecht, 1994, xi + 324 pp.

Throughout the history of applied mathematics, investigators have found integral transforms to be of great use in solving practical problems. An integral transform is a mapping  $f \mapsto g$  defined by

$$g(y) = \int_0^\infty k(x, y) f(x) \, dx,$$

where k(x, y) is called the kernel of the transform. It was observed that many integral transforms possess an inversion formula of the type

$$f(x) = \int_0^\infty k^*(x, y) g(y) \, dy.$$

(Sometimes the path of integration in the second integral is complex.) The two formulas above constitute an integral transform pair, usually named after the inventor.

The authors of this treatise study two sorts of transforms. The first they call Mellin convolution type transforms. Among the best known of these are  $k(x, y) = e^{-xy}$  (Laplace),  $k(x, y) = x^{y-1}$  (Mellin),  $k(x, y) = \cos xy$  (Fourier cosine),  $k(x, y) = \sqrt{xy} K_y(xy)$  (Meijer),  $k(x, y) = (xy)^{\mu-1/2} e^{-xy/2} W_{\kappa,\mu}(xy)$  (generalized Meijer), and  $k(x, y) = (x + y)^{-\rho}$  (Stieltjes). The reader will observe that all these kernels are hypergeometric type functions—indeed, any other sort of kernel is virtually unknown. The inversion formulas for these transforms are known and their kernels are closely related hypergeometric functions. Further, integration in both the transform and its inversion formula are conducted with respect to the argument of the hypergeometric function.

The second type are what the authors call index transforms. Again, these transforms have kernels of hypergeometric type, but integration in the inversion formula is performed with respect to a parameter of a hypergeometric function. Two examples are the transform pairs

$$g(y) = \frac{2}{\pi^2} y \sinh \pi y \int_0^\infty \frac{K_0(x)}{x} f(x) \, dx, \qquad f(x) = \int_0^\infty K_{iy}(x) \, g(y) \, dy,$$

(Lebedev), and

$$g(y) = \Gamma(\frac{1}{2} - \kappa - iy) \Gamma(\frac{1}{2} - \kappa + iy) \int_0^\infty W_{\kappa, iy}(x) f(x) dx$$
$$f(x) = \frac{1}{(x\pi)^2} \int_0^\infty y \sinh(2\pi y) W_{\kappa, iy}(x) g(y) dy,$$

(Wimp).

The authors of this very welcome book give the first detailed treatment of integral transforms with hypergeometric kernels. They discuss conditions for the validity of inversion formulas, generalized notions of convolution, Parseval type equalities, Erdélyi-Kober fractional operators, convolutional rings, and the application of transforms to the evaluation of integrals and to the solution of integral equations. The book fills a regrettable gap in the mathematical literature. Since Titchmarsh's rather cursory treatment of integral transforms in the book *Theory of Fourier Integrals*, we have lacked any systematic exposition of these exciting and useful ideas.

Jet Wimp

G. V. MILOVANOVIĆ, D. S. MITRINOVIĆ, AND TH. M. RASSIAS, Topics in Polynomials: Extremal Problems, Inequalities, Zeros, World Scientific, Singapore, 1994, xiii + 821 pp.

"Polynomials pervade mathematics and much that is beautiful in mathematics is related to polynomials. Virtually every branch of mathematics from algebraic number theory and algebraic geometry to applied analysis, Fourier analysis, and computer science has its corpus of theory arising from the study of polynomials. Historically questions relating to polynomials, for example, the solution of polynomial equations, gave rise to some of the most important problems of the day. The subject is now much too large to attempt an encyclopedic coverage."\*

The body of the material the authors selected to explore, focuses on extremal problems and inequalities for polynomials, and properties of the zeros of polynomials. This is a book about classical algebraic and trigonometric polynomials. The discussion does not treat polynomials in an extended sense, and does not cover topics like Chebyshev, Markov, or Descartes systems, Müntz polynomials (or equivalently exponential sums), or rational function spaces. Some classical subjects, such as orthogonal polynomials, are not studied either, partly because their discussion would require separate books, and partly because such books exist.

In the preface the authors write: "The present book contains some of the most important results on the analysis of polynomials and their derivatives. Besides the fundamental results, which are treated with their proofs, the book also provides an account of the most recent developments concerning extremal properties of polynomials and their derivatives, as well as properties of their zeros. An attempt has been made to present the material in an integrated and selfcontained fashion. The book is intended not only for the specialist mathematician, but also for those researchers in the applied and computational sciences who use polynomials as a tool."

The topics are tastefully selected and the results are easy to find. Although this book is not really planned as a textbook to teach from, it is excellent for self-study or seminars. This is

\* From the preface of the book "Polynomials and Polynomial Inequalities" by P. Borwein and T. Erdélyi.

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